# Agent Interpolation in Distributed Systems

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(joint work with Marta Bílková and Wesley Fussner)

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**Main result**

We have constructively proved agent + Lyndon interpolation for $\mathsf{S5}_n$, multi-agent logic of knowledge in distributed systems.

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If $S5_n \vdash A \to B$, there exists an interpolant $C$ such that

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|---|---|---|---|
| every positive atom $p$ | | occurs positively | |
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Logic of Kripke frames with equivalence relation $\sim_i$ for each agent.

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## Clarification 2: $\mathsf{S5}_n$

Logic of Kripke frames with equivalence relation $\sim_i$ for each agent.

## Clarification 3: constructively

We designed cut-free crossword-sequent proof calculus allowing to construct interpolant $C$.

# Why Interpolation?

## Theory and Applications of Craig Interpolation (ed. ten Cate et al.)

### Contents

# Interpolation

**General shape of interpolation for a logic L (excluding uniform)**

If $B$ is a consequence of $A$ according to L, then there is an interpolant $C$ such that

- $C$ is a consequence of $A$ according to L,
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**Some options for consequence according to L**

- $\mathsf{L} \vdash A \to B \quad \Rightarrow \quad \mathsf{L} \vdash A \to C \,\&\, \mathsf{L} \vdash C \to B$ Craig, Lyndon
- $A \vdash_{\mathsf{L}} B \quad \Rightarrow \quad A \vdash_{\mathsf{L}} C \vdash_{\mathsf{L}} B$ deductive

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**Some options for common language**

- common atoms — Craig
- common positive and negative atoms — Lyndon
- common modalities — strong or multimodal or agent

# Interpolation

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## In this talk

- $\mathsf{L} \vdash A \to B \quad \Rightarrow \quad \mathsf{L} \vdash A \to C \;\&\; \mathsf{L} \vdash C \to B$ Lyndon

## In this talk

- common positive and negative atoms Lyndon
- common modalities agent

# Agent Interpolation Is Not Trivial

**Example (Marx, 1999)**

Consider logic $\mathsf{L} := \mathsf{K}_2 + \Box_1 A \to \Box_2 A$

Then

$$\mathsf{L} \vdash \Box_1 p \to \Box_2 p$$

but agent interpolants have to be purely propositional, which is not possible.

# Logic of Knowledge S5

## Hilbert axioms

- propositional tautologies
- $K(A \to B) \to (KA \to KB)$ — logical omniscience 1
- $KA \to A$ — factivity
- $KA \to KKA$ — positive introspection
- $\neg KA \to K\neg KA$ — negative instrospection
- from $A$ and $A \to B$, infer $B$ — modus ponens
- from $A$, infer $KA$ — necessitation, logical omniscience 2

## Semantics

$\mathsf{S5} \vdash A$

iff

$A$ is valid in all Kripke models $(W, \sim, V)$

with $\sim$ an equivalence relation.

# Why Is That Knowledge?

## Distributed systems

Multi-agent systems where several agents aim to fulfill a common task, but each has independent control and coordination requires communication.

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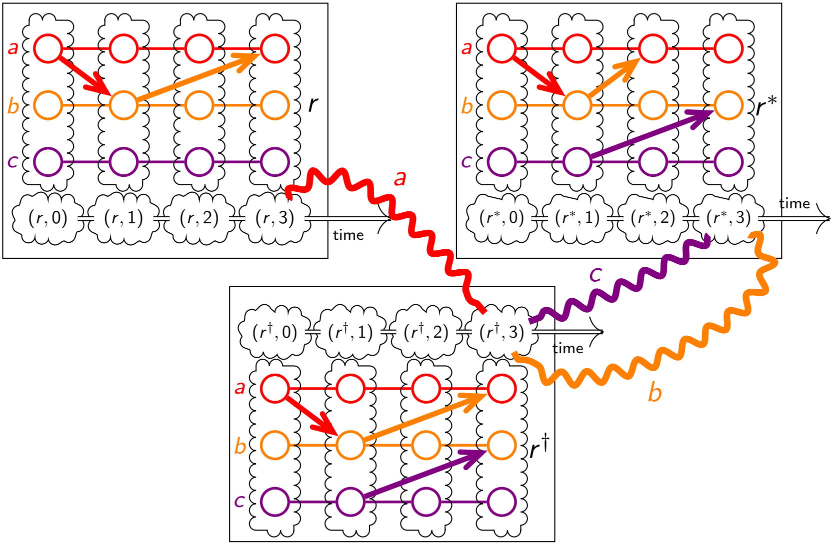
## Runs-and-systems paradigm

- a world is a global state $(r, t)$ of a run $r$ at time $t$
- in global state $(r, t)$, each agent $i$ has local state $r_i(t)$
- global states $(r, t) \sim_i (r', t')$ are indistinguishable by agent $i$ iff it has the same local state $r_i(t) \neq r'_i(t')$
- such $\sim_i$ is necessarily an equivalence relation

# Modeling Knowledge in Distributed Systems



# $S5_n$, Logic of Knowledge in Distributed Systems

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## Semantics

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iff

$A$ is valid in all Kripke models $(W, \sim_1, \dots, \sim_n, V)$

with each $\sim_i$ an equivalence relation.

# What is known

- S5 has Lyndon (hence, Craig) interpolation
- Fusion preserves Craig interpolation (Kracht–Wolter, 1991)
- $S5_n$ has Lyndon interpolation
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**One method to prove them all?**

We say: proof theory!

# Proof-theoretic method is constructive

(rules and conditions apply)

**Van Benthem's proof does not yield an efficient* method of finding interpolants**

* For any recursively enumerable logic, any proof that an interpolant exists yields an algorithm for finding an interpolant by enumerating all theorems of the logic. Mattias Baaz

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**Proof-theoretic method provides an efficient* method for constructing an interpolant**

* The interpolant is constructed not from the given valid implication $A \to B$ but rather from its derivation in a suitable proof calculus, e.g., from a cut-free sequent derivation of $A \Rightarrow B$.

# Reviewer 2: Begone, 'tis Trivial to Go from S5 to $S5_n$

**Authors 1, 2, 3**

It is only trivial if one does not try.

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**Cut-free hypersequents for S5**

- Mints, 1968; Pottinger, 1983; Avron, 1996
- Hypersequents are not trivial to extend to many modalities.
- They rely on all worlds being accessible from each other. This restriction works for S5 but not for S5$_n$.

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### Cut-free nested sequents for S5

- Brünnler, 2006; Poggiolesi, 2007
- Extended to S5$_n$ (Girlando–Lellmann–Olivetti, 2019)
- Works the way one expects
- I dare anyone to claim that nested cut elimination is trivial.

# Our Goal: Efficient, Minimalistic Proof Calculus for $S5_n$

## Analogy with S5

- Hypersequents correspond to simplified prefixed tableaux, i.e., with integer prefixes 1, 2, 3, ...
- Nested sequents correspond to prefixed tableaux where prefixes are sequences of integers 1, 1.1, 1.2, 1.1.1, 1.1.2, ...

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We wanted sequents for $S5_n$ with structure as simple as possible.

# Main Intuition

- Models for $S5_n$ consist of intersecting clusters of worlds.
- Each $\Box_i$ behaves like S5-modality within its $i$-clusters.
- Alternation $\Box_i \Box_j$ behaves like

  T-modalities switching from an $i$-cluster to the $j$-cluster.
- So our proof calculus combines

hypersequent S5 and nested sequent T features,
with any two hypersequent clusters intersecting
in at most one component,
like a (multi-dimensional) crossword.

# Crossword Derivation of $S5_2 \vdash \Diamond_2\Diamond_1\Box_1\Diamond_2 p \to \Diamond_2 p$

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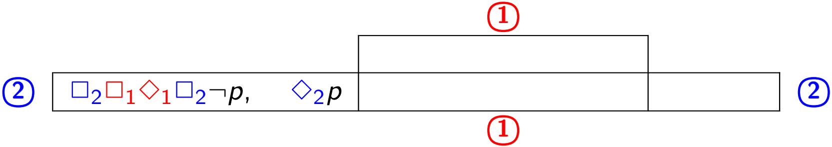

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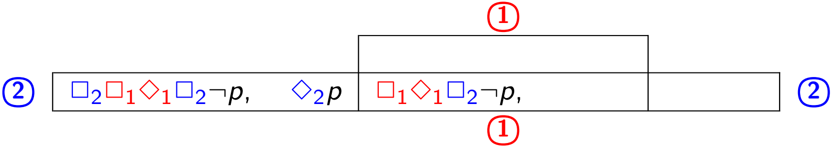

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|---|---|---|---|---|
| | | ① | | |
| | | $\Diamond_1\Box_2\neg p$ | | |
| ② | $\Box_2\Box_1\Diamond_1\Box_2\neg p$, $\Diamond_2 p$ | $\Box_1\Diamond_1\Box_2\neg p$, | | ② |
| | | ① | | |

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| | | ① | | |

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| | | | | |
|---|---|---|---|---|
| | | (1) | | |
| | | $\Diamond_1\Box_2\neg p$ | | |
| (2) | $\Box_2\Box_1\Diamond_1\Box_2\neg p, \quad \Diamond_2 p$ | $\Box_1\Diamond_1\Box_2\neg p, \quad \Box_2\neg p$ | $\neg p,$ | (2) |
| | | (1) | | |

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| | | ① | | |

# Crossword Sequent Calculus for S5$_n$

$$\frac{}{\mathcal{S}\{p, \neg p\}} \qquad \frac{}{\mathcal{S}\{\top\}}$$

$$\frac{\mathcal{S}\{A \vee B, A, B\}}{\mathcal{S}\{A \vee B\}} \qquad \frac{\mathcal{S}\{A \wedge B, A\} \quad \mathcal{S}\{A \wedge B, B\}}{\mathcal{S}\{A \wedge B\}}$$

$$\frac{\mathcal{S}\{[A|\Box_i A, \mathcal{T}|H]_i\}}{\mathcal{S}\{[\Box_i A, \mathcal{T}|H]_i\}} \qquad \frac{\mathcal{S}\{\Box_i A, [A|H]_i\}}{\mathcal{S}\{\Box_i A, [H]_i\}} \qquad \frac{\mathcal{S}\{\Box_i A, [A]_i\}}{\mathcal{S}\{\Box_i A\}}\ \Box_i A \not\leadsto i$$

$$\frac{\mathcal{S}\{[\Diamond_i A, \mathcal{T}|A, \mathcal{U}|H]_i\}}{\mathcal{S}\{[\Diamond_i A, \mathcal{T}|\mathcal{U}|H]_i\}} \qquad \frac{\mathcal{S}\{\Diamond_i A, [A, \mathcal{T}|H]_i\}}{\mathcal{S}\{\Diamond_i A, [\mathcal{T}|H]_i\}}$$

$$\frac{\mathcal{S}\{\Diamond_i A, A\}}{\mathcal{S}\{\Diamond_i A\}} \qquad \frac{\mathcal{S}\{A, \mathcal{T}, [\Diamond_i A, \mathcal{U}|H]_i\}}{\mathcal{S}\{\mathcal{T}, [\Diamond_i A, \mathcal{U}|H]_i\}}$$

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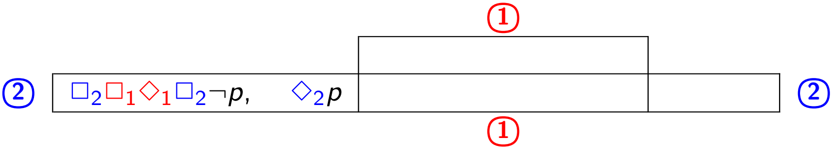

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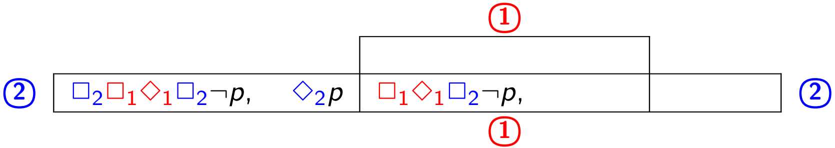
$$\dfrac{\dfrac{\Box_2\Box_1\Diamond_1\Box_2\neg p,\ \Diamond_2 p,\quad \left[\Box_1\Diamond_1\Box_2\neg p\right]_2}{\Box_2\Box_1\Diamond_1\Box_2\neg p,\ \Diamond_2 p}}{\Box_2\Box_1\Diamond_1\Box_2\neg p \lor \Diamond_2 p}$$



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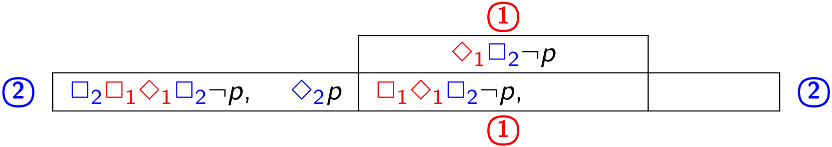

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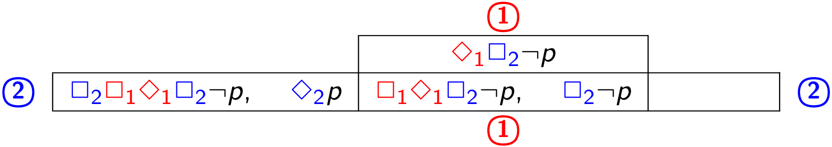

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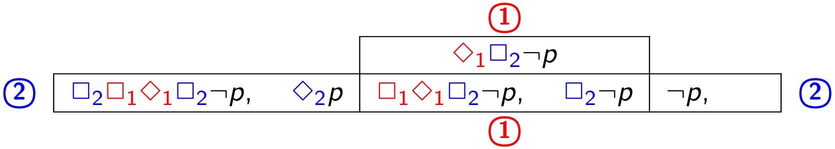

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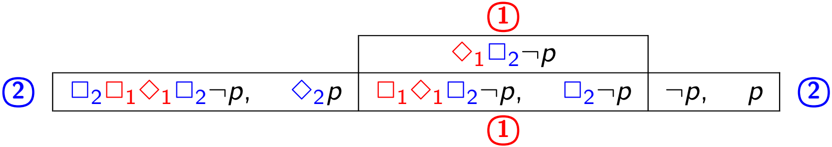
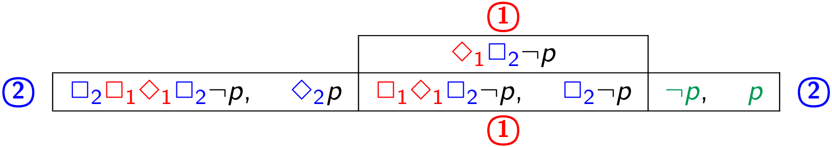


$$
\begin{array}{c}
\Box_2\Box_1\Diamond_1\Box_2\neg p,\ \Diamond_2 p,\quad \left[\Box_1\Diamond_1\Box_2\neg p,\ \Box_2\neg p,\ [\Diamond_1\Box_2\neg p]_1 \ \Big|_2 \ \neg p,\ p\right]_2 \\
\hline
\Box_2\Box_1\Diamond_1\Box_2\neg p,\ \Diamond_2 p,\quad \left[\Box_1\Diamond_1\Box_2\neg p,\ \Box_2\neg p,\ [\Diamond_1\Box_2\neg p]_1 \ \Big|_2 \ \neg p\right]_2 \\
\hline
\Box_2\Box_1\Diamond_1\Box_2\neg p,\ \Diamond_2 p,\quad \left[\Box_1\Diamond_1\Box_2\neg p,\ \Box_2\neg p,\ [\Diamond_1\Box_2\neg p]_1\right]_2 \\
\hline
\Box_2\Box_1\Diamond_1\Box_2\neg p,\ \Diamond_2 p,\quad \left[\Box_1\Diamond_1\Box_2\neg p,\ [\Diamond_1\Box_2\neg p]_1\right]_2 \\
\hline
\Box_2\Box_1\Diamond_1\Box_2\neg p,\ \Diamond_2 p,\quad \left[\Box_1\Diamond_1\Box_2\neg p\right]_2 \\
\hline
\Box_2\Box_1\Diamond_1\Box_2\neg p,\ \Diamond_2 p \\
\hline
\Box_2\Box_1\Diamond_1\Box_2\neg p \lor \Diamond_2 p
\end{array}
$$



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1. Sound and complete for $S5_n$
2. Hence, weakening, contraction, and cut admissibility
3. All rules are invertible
4. Strong termination (if redundant uses are prohibited)
5. Countermodel extraction from any leaf of a failed proof
6. Hence, decidability and finite model property

# Results about Interpolation

**Without boring you with the details**

- Maehara's method for sequents
- Fitting–K's method for generalized sequents

can be adapted to crossword sequents.

**Theorem**

*Given a crossword sequent derivation of $A \Rightarrow B$, there is an algorithm constructing an agent + Lyndon interpolant of $A \to B$.*

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Thank you!

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