Kamp Theorem for Pomset Languages of Higher Dimensional Automata

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Context

Finite-state automata

Higher-dimensional automata[1]

[1] Vaughan Pratt, "Modeling concurrency with geometry", POPL'91

[2] Fahrenberg et al., "Languages of Higher-Dimensional Automata",2021

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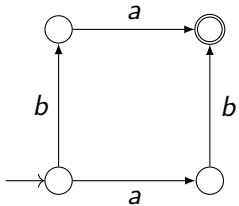
Finite-state automata

Interleaving



Accepts **words**
= **Totally** ordered multisets

$\{a \to b, b \to a\}$

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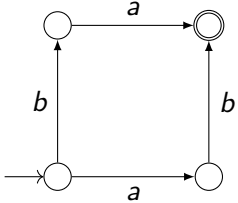

Accepts **words**
= **Totally** ordered multisets

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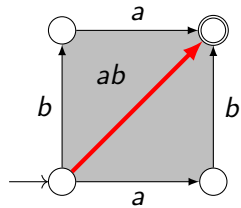
Higher-dimensional automata[1]

True concurrency



Accepts **pomsets**[2]
= **Partially** ordered multisets

$\{\begin{smallmatrix} a \\ b \end{smallmatrix}, a \to b, b \to a\}$

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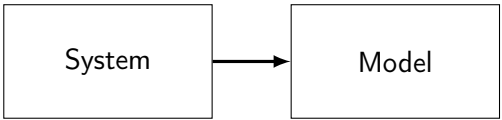
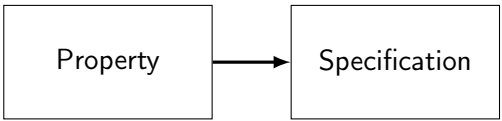
Model-checking





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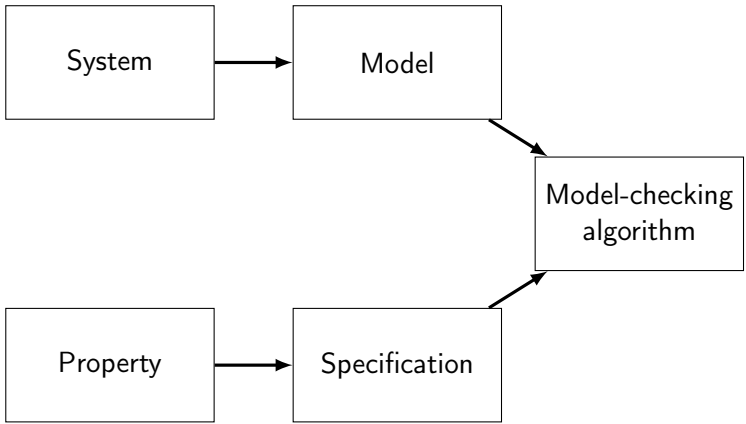

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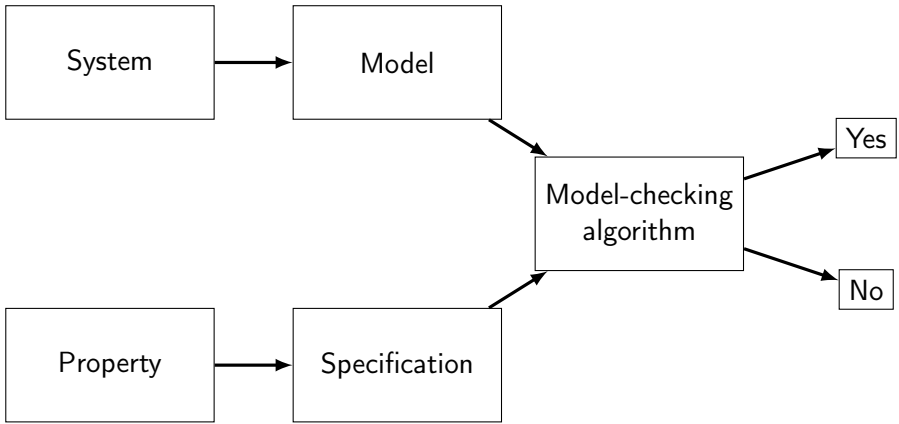

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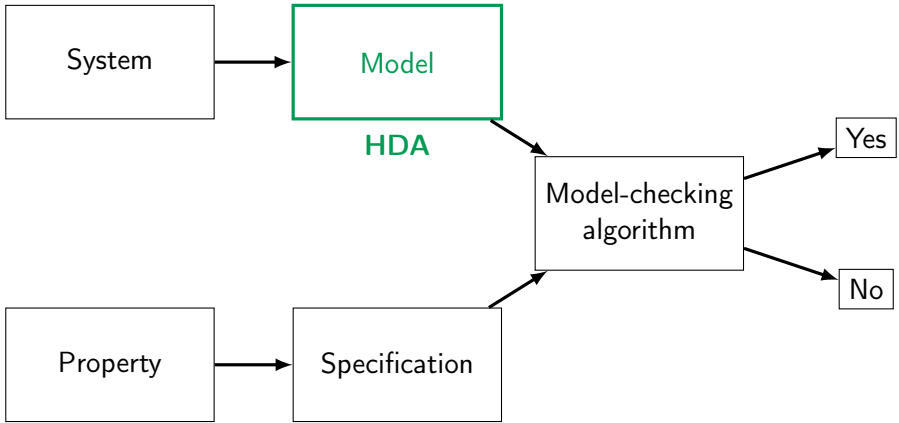

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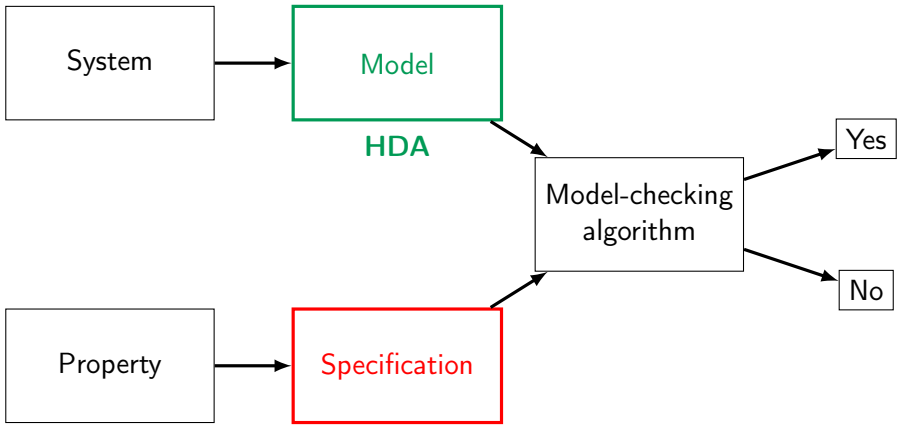
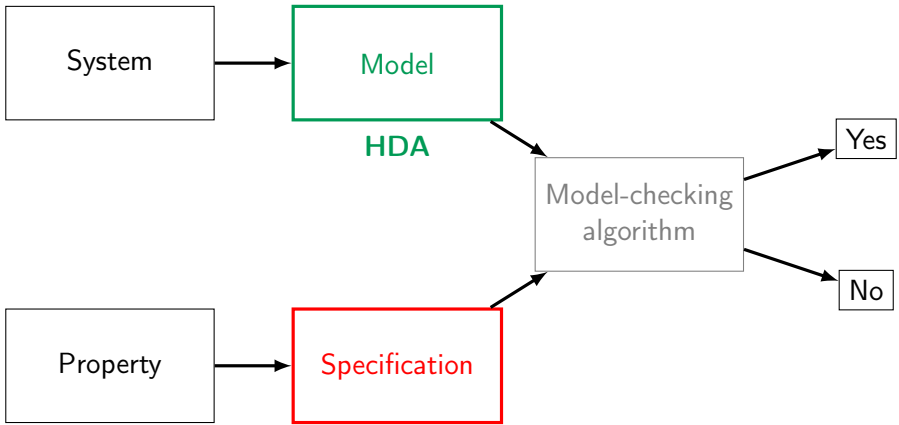


This talk: define LTL for HDAs

Model-checking

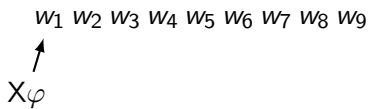


This talk: define LTL for HDAs

Reminder: LTL over words

$w_1\ w_2\ w_3\ w_4\ w_5\ w_6\ w_7\ w_8\ w_9$

Reminder: LTL over words



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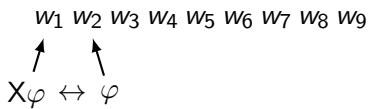

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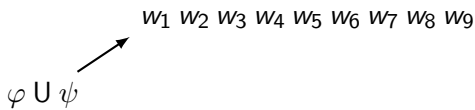

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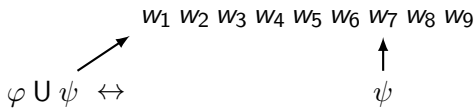

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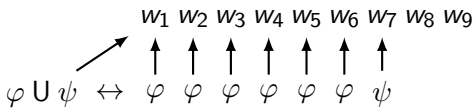

Kamp's theorem (1968)

Over words, **FO** = **LTL**

Examples

Kamp's theorem (1968)

Over words, **FO = LTL**

Examples

- Every a is finally followed by a b:

  FO: $\forall x, a(x) \Rightarrow \exists y, x < y \wedge b(y)$

  LTL: $\mathsf{G}(a \Rightarrow \mathsf{F}b)$

Kamp's theorem (1968)

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- There is no b happening before the first a (every b is preceded by an a):

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What about pomsets?

State of the art

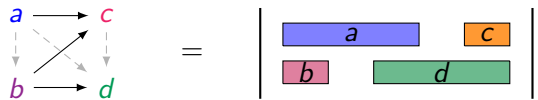
[Diekert and Gastin 2006] FO $\equiv$ LTL-style logics over **Mazurkiewicz** traces

[Johansen 2014] Defines modal logic for **higher-dimensional structures**

[Zouari et al. 2024] Defines **CTL-like** (branching time) logic for HDA

[Amrane et al. 2024] **Monadic second order** (MSO) logic $\equiv$ HDAs

Interval partially ordered multiset with interfaces (Pomsets)



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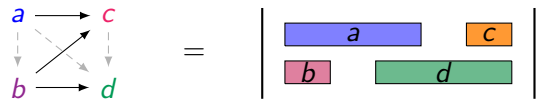


Formally, given an alphabet Σ:

P set of events

$<_P$ precedence order (represented by $\rightarrow$),

$\dashrightarrow_P$ event order (represented by $\dashrightarrow$, or top-to-bottom),

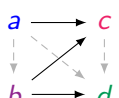
$\lambda_P : P \to \Sigma$ labelling function.

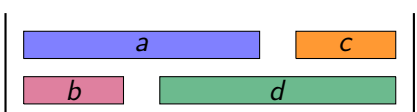
Naïve attempt

Syntax

- **Next:** $\langle \rightarrow \rangle$
- **Event order:** $\langle \dashrightarrow \rangle, \langle \dashrightarrow^{-1} \rangle$
- **Until:** U

$a \models \langle \rightarrow \rangle \langle \dashrightarrow \rangle d$





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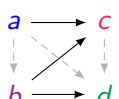
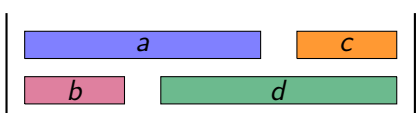
$$\text{iff } c \models \langle \dashrightarrow \rangle\, d$$





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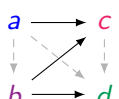

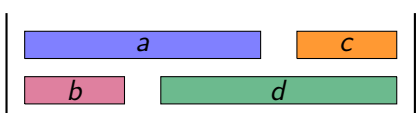

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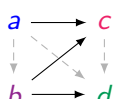

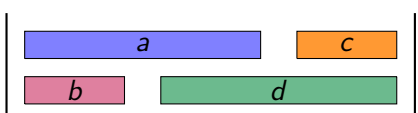

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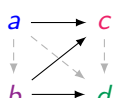

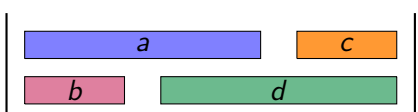

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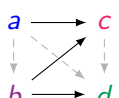

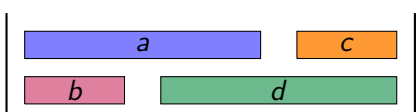

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$$
\begin{aligned}
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\text{iff } d &\models \langle \dashrightarrow^{-1} \rangle a \\
\text{if } a &\models a
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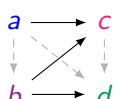
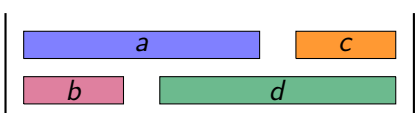
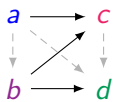
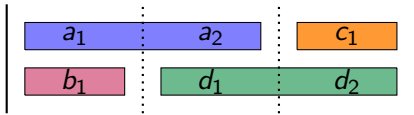




Sub-events





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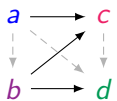
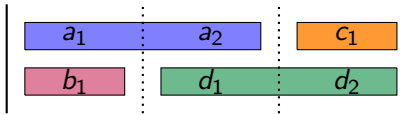
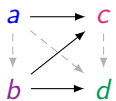


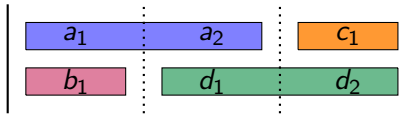


Example

- $a_1 = (a, a)$
- $a_2 = (a, d)$

Sub-events





Example

- $a_1 = (a, a)$
- $a_2 = (a, d)$

Definition

Sub-event (e, m):

- e = the event
- m = most recently started event

Linear Temporal Logic for pomsets

Fix a finite alphabet Σ

Syntax

$$\varphi, \psi ::= a \mid \mathsf{s} \mid \mathsf{t} \mid \neg\varphi \mid \varphi \wedge \psi \mid \langle \rightarrow \rangle \varphi \mid \langle \dashrightarrow \rangle \varphi \mid \langle \dashrightarrow^{-1} \rangle \varphi \mid \varphi \,\mathsf{U}\, \psi$$

With $a \in \Sigma$

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- **Next sub-event:** $\langle \rightarrow \rangle$
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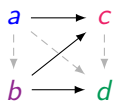
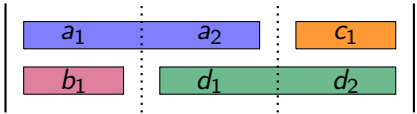
- **Until:** U
- **Starts:** s
- **Terminates:** t

Semantics – Atomic formulas





- **Letters** $a \in \Sigma$: $a_2 \models a$

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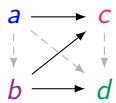
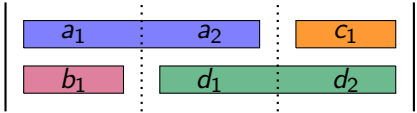
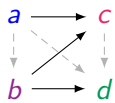
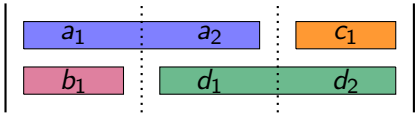




- **Letters** $a \in \Sigma$: $a_2 \models a$
- **Starts/terminates:** $d_1 \models \mathsf{s} \wedge \neg \mathsf{t}$

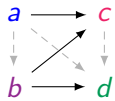
Semantics – Next

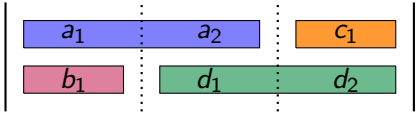




- $a_1 \not\models \langle\rightarrow\rangle\, c$

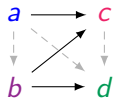
Semantics – Next

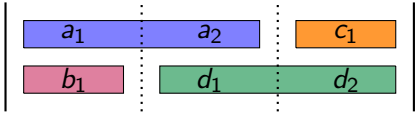




- $a_1 \not\models \langle\rightarrow\rangle\, c$ because $a_1 \rightarrow a_2$ and $a_2 \not\models c$

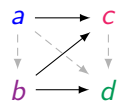
Semantics – Next

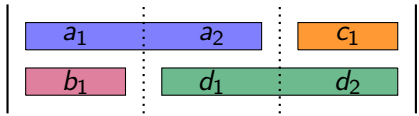




- $a_1 \not\models \langle\rightarrow\rangle\, c$ because $a_1 \rightarrow a_2$ and $a_2 \not\models c$
- $a_1 \models \langle\rightarrow\rangle\, \langle\rightarrow\rangle\, c$

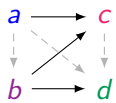
Semantics – Next

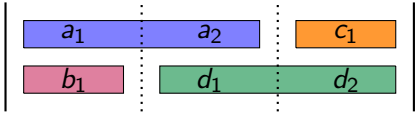




- $a_1 \not\models \langle\rightarrow\rangle\, c$ because $a_1 \rightarrow a_2$ and $a_2 \not\models c$
- $a_1 \models \langle\rightarrow\rangle\, \langle\rightarrow\rangle\, c$ because $a_1 \rightarrow a_2 \rightarrow c_1$ and $c_1 \models c$

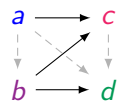
Semantics – Next

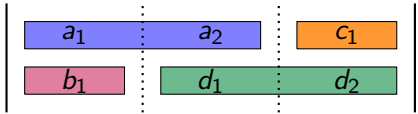




- $a_1 \not\models \langle\rightarrow\rangle\, c$ because $a_1 \rightarrow a_2$ and $a_2 \not\models c$
- $a_1 \models \langle\rightarrow\rangle\, \langle\rightarrow\rangle\, c$ because $a_1 \rightarrow a_2 \rightarrow c_1$ and $c_1 \models c$
- $a_2 \not\models \langle\rightarrow\rangle\, d$

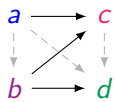
Semantics – Next

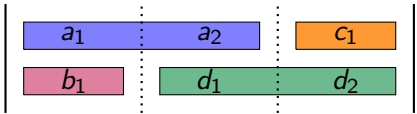




- $a_1 \not\models \langle\rightarrow\rangle\, c$ because $a_1 \rightarrow a_2$ and $a_2 \not\models c$
- $a_1 \models \langle\rightarrow\rangle\langle\rightarrow\rangle\, c$ because $a_1 \rightarrow a_2 \rightarrow c_1$ and $c_1 \models c$
- $a_2 \not\models \langle\rightarrow\rangle\, d$ because $a_2 \not\rightarrow d_1$

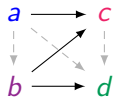
Semantics – Event order

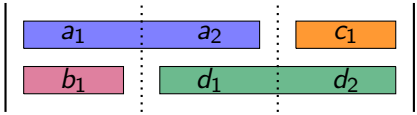




- $a_1 \models \langle \dashrightarrow \rangle\, b$ but $a_1 \not\models \langle \dashrightarrow \rangle\, d$
- $b_1 \models \langle \dashrightarrow^{-1} \rangle\, a$

Semantics – Event order



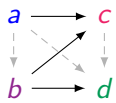


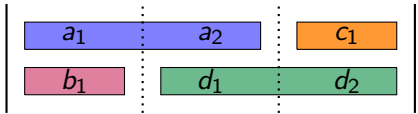
- $a_1 \models \langle \dashrightarrow \rangle\, b$ but $a_1 \not\models \langle \dashrightarrow \rangle\, d$
- $b_1 \models \langle \dashrightarrow^{-1} \rangle\, a$

Example

$$\langle \| \rangle\, a =$$

Semantics – Event order



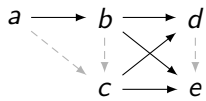


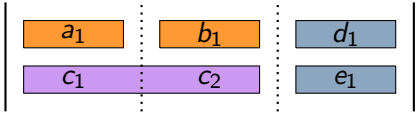
- $a_1 \models \langle \dashrightarrow \rangle\, b$ but $a_1 \not\models \langle \dashrightarrow \rangle\, d$
- $b_1 \models \langle \dashrightarrow^{-1} \rangle\, a$

Example

$$\langle \| \rangle\, a = \bigvee_{i=0}^{k} \langle \dashrightarrow \rangle^i a \vee \langle \dashrightarrow^{-1} \rangle^i a$$

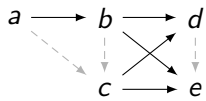
Semantics – Until

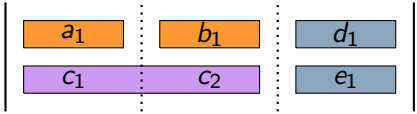




- $a_1 \models (a \vee b)\ \mathsf{U}\ e$

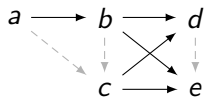
Semantics – Until

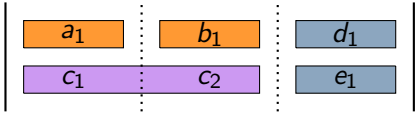




- $a_1 \models (a \vee b)\ \mathsf{U}\ e$ because $e_1 \models e$

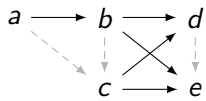
Semantics – Until

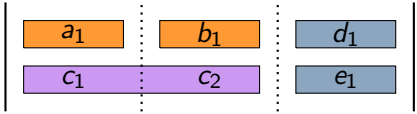




- $a_1 \models (a \vee b)\ \mathsf{U}\ e$ because $e_1 \models e$ and $a_1 \models a \vee b$

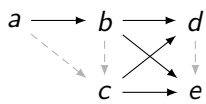
Semantics – Until

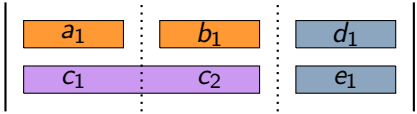




- $a_1 \models (a \vee b)\ \mathsf{U}\ e$ because $e_1 \models e$ and $a_1 \models a \vee b$ and $b_1 \models a \vee b$

Semantics – Until

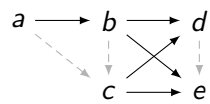


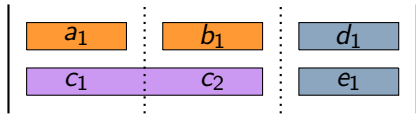


- $a_1 \models (a \vee b)\ \mathsf{U}\ e$ because $e_1 \models e$ and $a_1 \models a \vee b$ and $b_1 \models a \vee b$

- $a_1 \not\models (a \vee c)\ \mathsf{U}\ d$

Semantics – Until



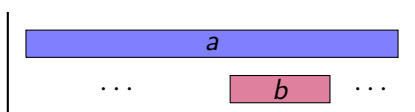


- $a_1 \models (a \vee b)\ \mathsf{U}\ e$ because $e_1 \models e$ and $a_1 \models a \vee b$ and $b_1 \models a \vee b$

- $a_1 \not\models (a \vee c)\ \mathsf{U}\ d$ because $a_1 \not\rightarrow c_2$

Example

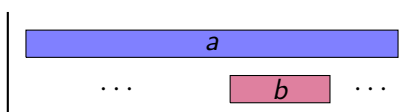
Goal: For any a, there is a b concurrent with a.



Example

Goal: For any a, there is a b concurrent with a.

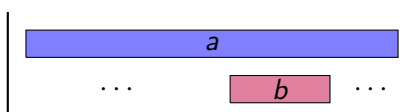
$$G[\ a \qquad \Rightarrow \qquad\qquad]$$



Example

Goal: For any a, there is a b concurrent with a.

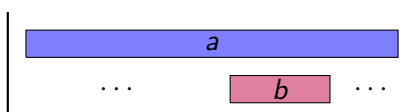
$$G[\ a \qquad \Rightarrow (\quad \mathsf{U}\ \langle\|\rangle\ b)]$$



Example

Goal: For any a, there is a b concurrent with a.

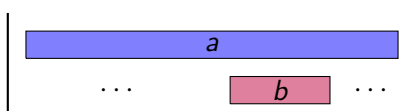
$$G[\ a \qquad \Rightarrow (\neg \mathrm{t}\ \mathsf{U}\ \langle\!|\!|\rangle\ b)]$$



Example

Goal: For any a, there is a b concurrent with a.

$$G[(a \wedge \mathsf{s}) \Rightarrow (\neg \mathsf{t}\ \mathsf{U}\ \langle\!|\!|\rangle\ b)]$$



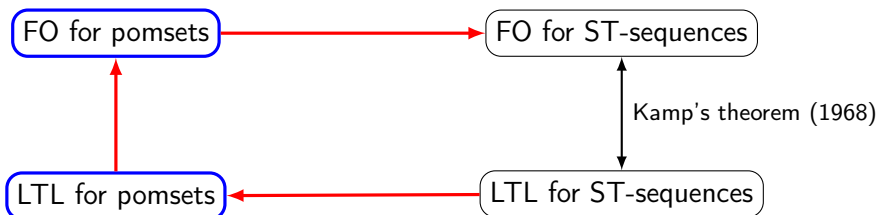
Expressive power

Over pomsets, **FO** = **LTL**

Expressive power

Over pomsets, **FO = LTL**

Proof sketch

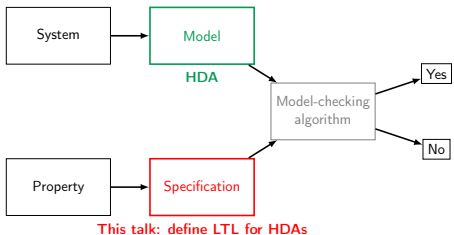


ST-sequences = word representation of pomsets

Conclusion

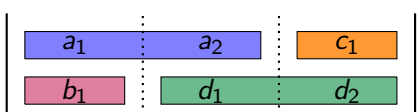
Contributions

- Sub-events
- LTL for pomsets
- LTL = FO



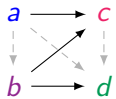
Future works

- Model-checking algorithms
- HDAs are too strict: partial HDAs?



ST-sequences

Example:

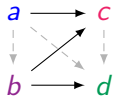


$\Rightarrow$ $\left[\begin{smallmatrix} a\bullet \\ \bullet b\bullet \end{smallmatrix}\right]\left[\begin{smallmatrix} \bullet a\bullet \\ \bullet b \end{smallmatrix}\right]\left[\begin{smallmatrix} \bullet a\bullet \\ d\bullet \end{smallmatrix}\right]\left[\begin{smallmatrix} \bullet a \\ \bullet d\bullet \end{smallmatrix}\right]\left[\begin{smallmatrix} c\bullet \\ \bullet d\bullet \end{smallmatrix}\right]\left[\begin{smallmatrix} \bullet c \\ \bullet d\bullet \end{smallmatrix}\right]$ (sparse)

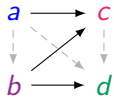
[3] Amrane et al., "Presenting Interval Pomsets with Interfaces", 2024

ST-sequences

Example:



$\Rightarrow$ $\left[\begin{smallmatrix} a\bullet \\ \bullet b\bullet \end{smallmatrix}\right]\left[\begin{smallmatrix} \bullet a\bullet \\ \bullet b \end{smallmatrix}\right]\left[\begin{smallmatrix} \bullet a\bullet \\ d\bullet \end{smallmatrix}\right]\left[\begin{smallmatrix} \bullet a \\ \bullet d\bullet \end{smallmatrix}\right]\left[\begin{smallmatrix} c\bullet \\ \bullet d\bullet \end{smallmatrix}\right]\left[\begin{smallmatrix} \bullet c \\ \bullet d\bullet \end{smallmatrix}\right]$ (sparse)

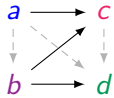


$\Rightarrow$ $[\bullet b\bullet]\left[\begin{smallmatrix} a\bullet \\ \bullet b\bullet \end{smallmatrix}\right]\left[\begin{smallmatrix} \bullet a\bullet \\ \bullet b \end{smallmatrix}\right]\left[\begin{smallmatrix} \bullet a\bullet \\ d\bullet \end{smallmatrix}\right]\left[\begin{smallmatrix} \bullet a \\ \bullet d\bullet \end{smallmatrix}\right]\left[\begin{smallmatrix} c\bullet \\ \bullet d\bullet \end{smallmatrix}\right]\left[\begin{smallmatrix} \bullet c\bullet \\ \bullet d\bullet \end{smallmatrix}\right]\left[\begin{smallmatrix} \bullet c \\ \bullet d\bullet \end{smallmatrix}\right]$ (non-sparse)

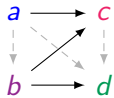
[3] Amrane et al., "Presenting Interval Pomsets with Interfaces", 2024

ST-sequences

Example:



$\Rightarrow$ $\left[\begin{smallmatrix} a\bullet \\ \bullet b\end{smallmatrix}\right]\left[\begin{smallmatrix} \bullet a\bullet \\ \bullet b\end{smallmatrix}\right]\left[\begin{smallmatrix} \bullet a\bullet \\ d\bullet\end{smallmatrix}\right]\left[\begin{smallmatrix} \bullet a \\ \bullet d\bullet\end{smallmatrix}\right]\left[\begin{smallmatrix} c\bullet \\ \bullet d\bullet\end{smallmatrix}\right]\left[\begin{smallmatrix} \bullet c \\ \bullet d\bullet\end{smallmatrix}\right]$ (sparse)



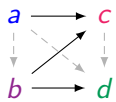
$\Rightarrow$ $[\bullet b\bullet]\left[\begin{smallmatrix} a\bullet \\ \bullet b\end{smallmatrix}\right]\left[\begin{smallmatrix} \bullet a\bullet \\ \bullet b\end{smallmatrix}\right]\left[\begin{smallmatrix} \bullet a\bullet \\ d\bullet\end{smallmatrix}\right]\left[\begin{smallmatrix} \bullet a \\ \bullet d\bullet\end{smallmatrix}\right]\left[\begin{smallmatrix} c\bullet \\ \bullet d\bullet\end{smallmatrix}\right]\left[\begin{smallmatrix} \bullet c\bullet \\ \bullet d\bullet\end{smallmatrix}\right]\left[\begin{smallmatrix} \bullet c \\ \bullet d\bullet\end{smallmatrix}\right]$ (non-sparse)

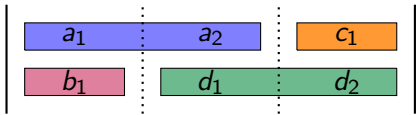
Properties:[3]

1. ST-sequences are finite **words** over a finite alphabet
2. Sparse ST-decompositions are **unique**
3. ST-sequences are **equivalent representations** of pomsets

[3] Amrane et al., "Presenting Interval Pomsets with Interfaces", 2024

LTL ST-seq $\to$ LTL pomsets – Invariant

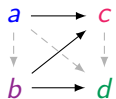


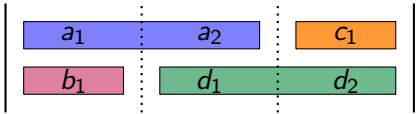


$\varphi \in \text{LTL ST-seq} \to \ulcorner\varphi\urcorner_{\text{s}}, \ulcorner\varphi\urcorner_{\text{t}} \in \text{LTL pomsets}$

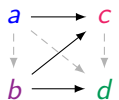
$$\begin{bmatrix} \bullet a \bullet \\ d \bullet \end{bmatrix} \cdots \models \varphi \iff a_2 \text{ and } d_1 \models \ulcorner\varphi\urcorner_{\text{s}}$$

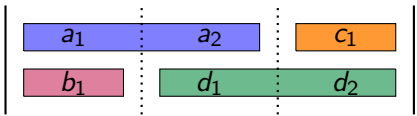
LTL ST-seq $\rightarrow$ LTL pomsets – Translation





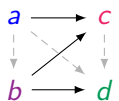
LTL ST-seq $\rightarrow$ LTL pomsets – Translation

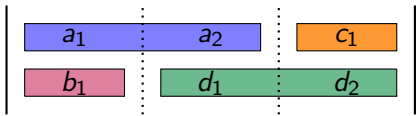




- $\ulcorner \mathsf{X}\psi \urcorner_{\mathsf{s}} = \ulcorner \psi \urcorner_{\mathsf{t}}$

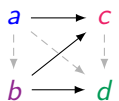
LTL ST-seq $\to$ LTL pomsets – Translation

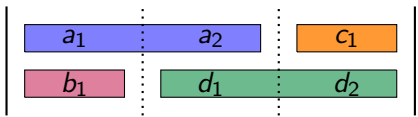




- $\ulcorner \mathsf{X}\psi \urcorner_{\mathsf{s}} = \ulcorner \psi \urcorner_{\mathsf{t}}$
- $\ulcorner \mathsf{X}\psi \urcorner_{\mathsf{t}} = \langle \| \rangle (\neg \mathsf{t} \wedge \langle \to \rangle \ulcorner \psi \urcorner_{\mathsf{s}}) = \mathsf{EX} \ulcorner \psi \urcorner_{\mathsf{s}}$

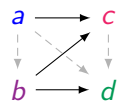
LTL ST-seq $\to$ LTL pomsets – Translation

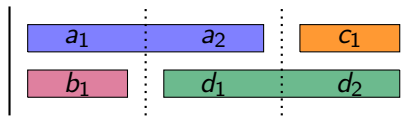




- $\ulcorner \mathsf{X}\psi \urcorner_{\mathsf{s}} = \ulcorner \psi \urcorner_{\mathsf{t}}$
- $\ulcorner \mathsf{X}\psi \urcorner_{\mathsf{t}} = \langle \| \rangle (\neg \mathsf{t} \wedge \langle \to \rangle \ulcorner \psi \urcorner_{\mathsf{s}}) = \mathsf{EX} \ulcorner \psi \urcorner_{\mathsf{s}}$
- $\ulcorner \psi_1 \,\mathsf{U}\, \psi_2 \urcorner_{\mathsf{s}} = (\ulcorner \psi_1 \urcorner_{\mathsf{s}} \wedge \ulcorner \psi_1 \urcorner_{\mathsf{t}}) \,\mathsf{U}\, (\ulcorner \psi_2 \urcorner_{\mathsf{s}} \vee (\ulcorner \psi_1 \urcorner_{\mathsf{s}} \wedge \ulcorner \psi_2 \urcorner_{\mathsf{t}}))$

LTL ST-seq $\to$ LTL pomsets – Translation





- $\ulcorner \mathsf{X}\psi \urcorner_{\mathsf{s}} = \ulcorner \psi \urcorner_{\mathsf{t}}$
- $\ulcorner \mathsf{X}\psi \urcorner_{\mathsf{t}} = \langle\!\|\rangle(\neg \mathsf{t} \wedge \langle\to\rangle \ulcorner \psi \urcorner_{\mathsf{s}}) = \mathsf{EX} \ulcorner \psi \urcorner_{\mathsf{s}}$
- $\ulcorner \psi_1 \,\mathsf{U}\, \psi_2 \urcorner_{\mathsf{s}} = (\ulcorner \psi_1 \urcorner_{\mathsf{s}} \wedge \ulcorner \psi_1 \urcorner_{\mathsf{t}}) \,\mathsf{U}\, (\ulcorner \psi_2 \urcorner_{\mathsf{s}} \vee (\ulcorner \psi_1 \urcorner_{\mathsf{s}} \wedge \ulcorner \psi_2 \urcorner_{\mathsf{t}}))$
- $\ulcorner \psi_1 \,\mathsf{U}\, \psi_2 \urcorner_{\mathsf{t}} = \ulcorner \psi_2 \urcorner_{\mathsf{t}} \wedge \mathsf{EX}((\ulcorner \psi_1 \urcorner_{\mathsf{s}} \wedge \ulcorner \psi_1 \urcorner_{\mathsf{t}}) \,\mathsf{U}\, (\ulcorner \psi_2 \urcorner_{\mathsf{s}} \vee (\ulcorner \psi_1 \urcorner_{\mathsf{s}} \wedge \ulcorner \psi_2 \urcorner_{\mathsf{t}})))$